Techno-pedagogic task design in a technology-rich mathematics classroom: the case of dynamic geometry

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A Technology-rich Mathematics Classroom

• Technology in a mathematics classroom serves as a pedagogical tool like ruler and compasses. It facilitates teachers and students to teach and to learn.
• Technology is at the same time consequence and driving force of human intellect. Thus, technology can be a means to acquire knowledge and may itself become part of knowledge.

Utilizing Information Communication Technology (ICT) in a mathematics classroom is not merely about presenting traditional school mathematics via new ICT media like Powerpoint; Internet or even sophisticated virtual environment like the dynamic geometry environment Sketchpad™; it is about how to harvest the power of the technology to create a new way of teaching, learning, and even thinking about mathematics.

In a technology-rich teaching and learning environment, the role of a teacher in the tradition sense must be cast-off in order to give room for students to discover and even to create knowledge. Teacher is to guide rather than to instruct, to suggest rather than to transmit. In this way, students could have ownership of the knowledge gained.

ICT should open a new space of learning that is broader in scope than the traditional classroom.

Mathematical Experience

Suitable ICT environments for mathematics learning have the power to allow students to conveniently make visible the different variations in a mathematical situation and to re-produce mental pictures that guide the development of mathematical concept (for example, in a dynamic geometry environment).

A mathematical experience can be seen as “the discernment of invariant pattern concerning numbers and/or shapes and the re-production or re-presentation of that pattern.” (Leung, 2010)


- An affordance in technology-rich teaching and learning environment is “the opportunity for interactivity between the user and the technology for some specific purpose” (Brown, 2005).
- Task design must consider how the affordance of a chosen ICT environment can facilitate or impede mathematical learning and how to capitalize it to enhance students’ ability to experience mathematics under an inquiry mode.

Leung, A. (2011, under revision) Techno-pedagogic task design in dynamic geometry environments, ZDM (Zentralblatt für Didaktik der Mathematik), 43 (4).

“A New Pedagogical Task Design Model

Task Design Principals for Technology-rich Mathematics Classroom

- Construct mathematical object using the technology involved
- Interact with the technology involved
- Observe and record
- Explain or prove
- Generalize findings into mathematical concepts
- Hypothesize and make conjecture

“Techno-pedagogic Task Design in Mathematics

Task design that focuses on pedagogical processes in which learners are empowered with amplified abilities to explore, re-construct (or re-invent) and explain mathematical concepts using tools embedded in a technology-rich environment.

“The purpose of a task is to initiate mathematically fruitful activity that leads to a transformation in what learners are sensitised to notice and competent to carry out” (Mason & Johnston-Wilder, 2006, p.25)

“The point of setting tasks for learners is to get them actively making sense of phenomena and exercising their powers and their emerging skills” (Mason & Johnston-Wilder, 2006, p.69)
Three nested epistemic task modes are put forward to guide the design of a techno-pedagogic task. They are:

- **Establishing Practices Mode**
- **Critical Discernment Mode**
- **Situated Discourse Mode**

Establishing Practices Mode (PM)

PM1
Construct mathematical objects or manipulate pre-designed mathematical objects using tools embedded in a technology-rich environment

PM2
Interact with the tools in a technology-rich environment to develop
(a) skill-based routines
(b) modalities of behaviour
(c) modes of situated dialogue

Establishing Situated Discourses Mode (SDM)

SD1
Develop inductive reasoning leading to making generalized conjecture

SD2
Develop discourses and modes of reasoning to explain or prove

Critical Discernment Mode (CDM)

Observe
Discover
Record

Re-present
Re-construct

Patterns of Variation and Invariant

An expanding pedagogic task exploration space in a technology-rich environment with a nested structure sequenced by the three epistemic modes

Dynamic Geometry Task Designs: Exploring Cyclic Quadrilateral
Design One

1. Construct a circle and four points on it
2. Join the four points with line segments to form a quadrilateral
3. Measure the four interior angles of the quadrilateral
4. Drag the four points to different positions on the circle
5. Investigate and make conjecture on the relationship among the angles
6. Explain (or prove) why the conjecture is true

Design Two

1. Construct a general quadrilateral ABCD
2. Measure two opposite interior angles, say $\angle ABC$ and $\angle CDA$
3. Calculate $\angle ABC + \angle CDA$
4. Turn the Trace function on for point C
5. Drag point C continuously to keep $\angle ABC + \angle CDA$ as close to 180° as possible
6. Observe the shape of the path that point C traces out
7. Make a conjecture on the shape of the path
8. Explain (or prove) why the conjecture is true

Design Three

Task 1

1. Construct two points A and B
2. Explore how to construct a circle that passes through A and B
3. Investigate how many such circles can be constructed
4. Explain why the construction procedure works

Task 2

1. Construct three non-collinear points A, B and C
2. Join A, B and C with line segments to form a triangle ABC
3. Explore how to construct a circle that passes through the vertices of ABC
4. Explain why the construction procedure works

Possible Conjectures

Design One:
Given a cyclic quadrilateral, a pair of interior opposite angles always adds up to 180°.

Design Two:
For a quadrilateral to satisfy the condition “a pair of interior opposite angles adds up to 180°”, the vertices of the quadrilateral must lie on a circle.

Design Three:
If the four perpendicular bisectors of the sides of a quadrilateral are concurrent, then the quadrilateral can be inscribed in a circle (see picture below).